

Feel Yourself a Student!

Dear friends, I would like to give to you an interesting and reliable antenna theory. Hours searching in the web gave me lots theoretical information about antennas. Really, at first I did not know what information to choose for ANTENTOP. Finally, I stopped on lectures "Modern Antennas in Wireless Telecommunications" written by Prof. Natalia K. Nikolova from McMaster University, Hamilton, Canada.

You ask me: Why?

Well, I have read many textbooks on Antennas, both, as in Russian as in English. So, I have the possibility to compare different textbook, and I think, that the lectures give knowledge in antenna field in great way. Here first lecture "Introduction into Antenna Study" is here. Next issues of ANTENTOP will contain some other lectures.

So, feel yourself a student! Go to Antenna Studies!

I.G.

*My Friends, the **above placed Intro** was given at ANTENTOP- 01- 2003 to Antennas Lectures.*

Now I know that the Lecture is one of popular topics of ANTENTOP. Every Antenna Lecture was downloaded more than 1000 times!

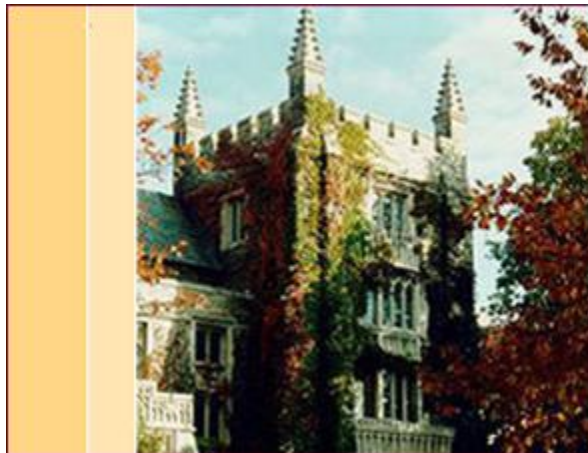
*Now I want to present to you one more very interesting Lecture 18- it is a Lecture **PLANAR ARRAYS, CIRCULAR ARRAYS**. I believe, you cannot find such info anywhere for free! Very interesting and very useful info for every ham, for every radio- engineer.*

So, feel yourself a student! Go to Antenna Studies!

I.G.

McMaster University Hall

Prof. Natalia K. Nikolova



PLANAR ARRAYS, CIRCULAR ARRAYS

Planar arrays are more versatile; they provide more symmetrical patterns with lower side lobes, much higher directivity (narrow main beam). They can be used to scan the main beam toward any point in space...

by Prof. Natalia K. Nikolova

LECTURE 18: PLANAR ARRAYS, CIRCULAR ARRAYS

1. Planar arrays

Planar arrays are more versatile; they provide more symmetrical patterns with lower side lobes, much higher directivity (narrow main beam). They can be used to scan the main beam toward any point in space.

Applications – tracking radars, remote sensing, communications, etc.

1.1 The array factor of a rectangular planar array

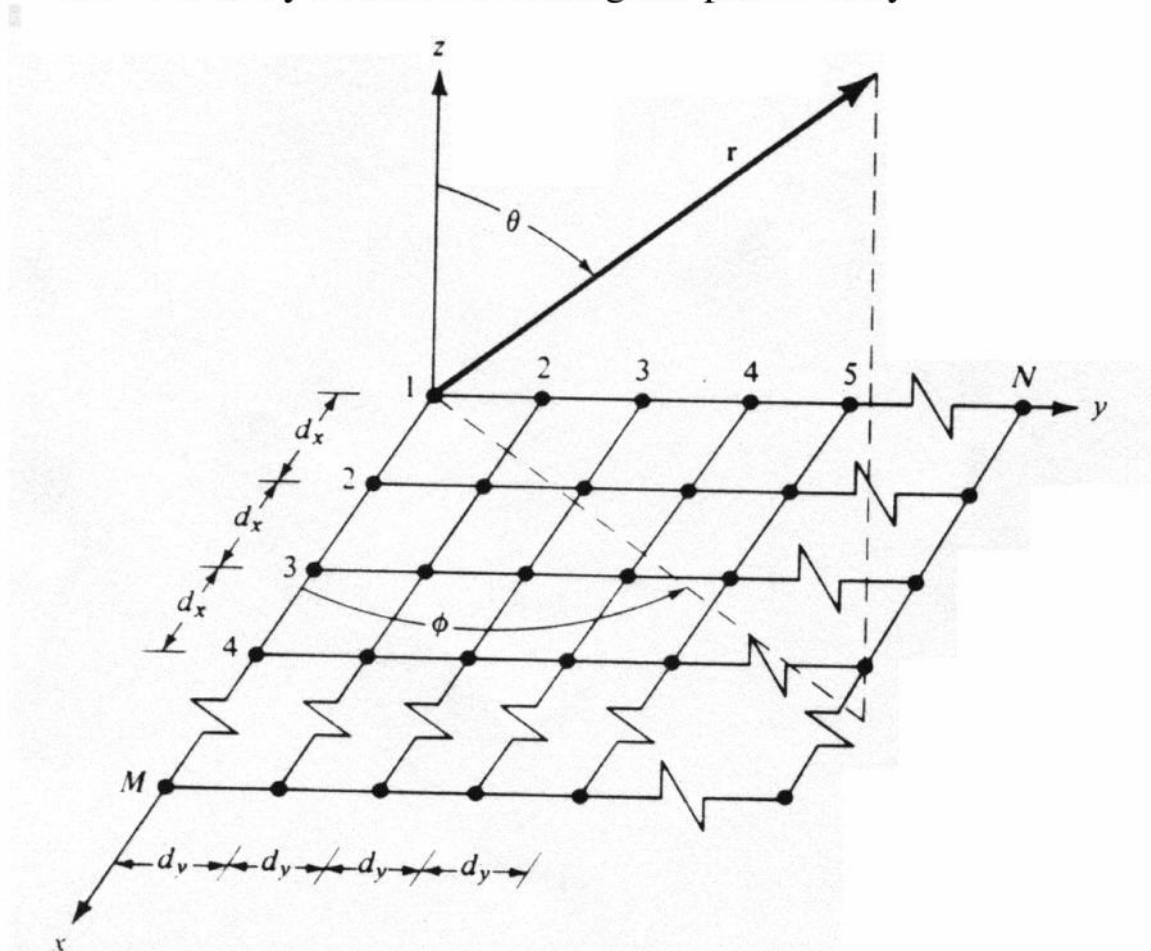


Fig. 6.23(b), pp.310, Balanis

The AF of a linear array of M elements along the x -axis is:

$$AF_{x1} = \sum_{m=1}^M I_{m1} e^{j(m-1)(kd \sin \theta \cos \phi + \beta_x)} \quad (18.1)$$

where $\sin \theta \cos \phi = \cos \gamma_x$ is the directional cosine with respect to the x -axis. It is assumed that all elements are equispaced with an interval of d_x and a progressive shift β_x . I_{m1} denotes the excitation amplitude of the element at the point with coordinates: $x = (m-1)d_x$, $y = 0$. In the figure above, this is the element of the m -th row and the 1st column of the array matrix.

If N such arrays are placed next to each other in the y direction, a rectangular array will be formed. We shall assume again that they are equispaced at a distance of d_y and there is a progressive phase shift along each row of β_y . It will be also assumed that the normalized current distribution along each of the x -directed array is the same but the absolute values correspond to a factor of I_{1n} ($n = 1, \dots, N$). Then, the AF of the entire array will be:

$$AF = \sum_{n=1}^N I_{1n} \left[\sum_{m=1}^M I_{m1} e^{j(m-1)(kd_x \sin \theta \cos \phi + \beta_x)} \right] e^{j(n-1)(kd_y \sin \theta \sin \phi + \beta_y)} \quad (18.2)$$

or

$$AF = S_{x_M} \cdot S_{y_N}, \quad (18.3)$$

where:

$$S_{x_M} = AF_{x1} = \sum_{m=1}^M I_{m1} e^{j(m-1)(kd \sin \theta \cos \phi + \beta_x)}, \text{ and}$$

$$S_{y_N} = AF_{1y} = \sum_{n=1}^N I_{1n} e^{j(n-1)(kd_y \sin \theta \sin \phi + \beta_y)}$$

In the array factors above:

$$\begin{aligned} \sin \theta \cos \phi &= \hat{x} \cdot \hat{r} = \cos \gamma_x \\ \sin \theta \sin \phi &= \hat{y} \cdot \hat{r} = \cos \gamma_y \end{aligned} \quad (18.4)$$

The pattern of a rectangular array is the product of the array factors of the linear arrays in the x and y directions.

For a uniform planar (rectangular) array $I_{m1} = I_{1n} = I_0$, for all m and n , i.e., all elements have the same excitation amplitudes.

$$AF = I_0 \sum_{n=1}^N e^{j(m-1)(kd_x \sin \theta \cos \phi + \beta_x)} \sum_{n=1}^N e^{j(n-1)(kd_y \sin \theta \sin \phi + \beta_y)} \quad (18.5)$$

The normalized array factor can be obtained as:

$$AF_n(\theta, \phi) = \left\{ \frac{1}{M} \frac{\sin\left(M \frac{\psi_x}{2}\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1}{N} \frac{\sin\left(N \frac{\psi_y}{2}\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right\}, \quad (18.6)$$

where:

$$\psi_x = kd_x \sin \theta \cos \phi + \beta_x$$

$$\psi_y = kd_y \sin \theta \sin \phi + \beta_y$$

The major lobe (principal maximum) and grating lobes of the terms:

$$S_{x_M} = \frac{1}{M} \frac{\sin\left(M \frac{\psi_x}{2}\right)}{\sin\left(\frac{\psi_x}{2}\right)} \quad (18.7)$$

$$S_{y_N} = \frac{1}{N} \frac{\sin\left(N \frac{\psi_y}{2}\right)}{\sin\left(\frac{\psi_y}{2}\right)} \quad (18.8)$$

are located at angles such that:

$$kd_x \sin \theta_m \cos \phi_m + \beta_x = \pm 2m\pi, \quad m = 0, 1, \dots \quad (18.9)$$

$$kd_y \sin \theta_n \sin \phi_n + \beta_y = \pm 2n\pi, \quad n = 0, 1, \dots \quad (18.10)$$

The principal maxima correspond to $m = 0, n = 0$.

In general, β_x and β_y are independent from each other. But, if it is required that the main beams of S_{x_M} and S_{y_N} intersect (which is usually the case), then the common main beam is in the direction:

$$\theta = \theta_0 \text{ and } \phi = \phi_0, m = n = 0 \quad (18.11)$$

If the principal maximum is specified by (θ_0, ϕ_0) , then the progressive phases β_x and β_y must satisfy:

$$\beta_x = -kd_x \sin \theta_0 \cos \phi_0 \quad (18.12)$$

$$\beta_y = -kd_y \sin \theta_0 \sin \phi_0 \quad (18.13)$$

When β_x and β_y are specified, the direction of the main beam can be found by simultaneously solving (18.12) and (18.13):

$$\tan \phi_0 = \frac{\beta_y d_x}{\beta_x d_y} \quad (18.14)$$

$$\sin \theta_0 = \pm \sqrt{\left(\frac{\beta_x}{kd_x}\right)^2 + \left(\frac{\beta_y}{kd_y}\right)^2} \quad (18.15)$$

The grating lobes can be located by substituting (18.12) and (18.13) in (18.9) and (18.10):

$$\tan \phi_{mn} = \frac{\sin \theta_0 \sin \phi_0 \pm n\lambda/d_y}{\sin \theta_0 \cos \phi_0 \pm m\lambda/d_x} \quad (18.16)$$

$$\sin \theta_{mn} = \frac{\sin \theta_0 \cos \phi_0 \pm m\lambda/d_x}{\cos \phi_{mn}} = \frac{\sin \theta_0 \sin \phi_0 \pm n\lambda/d_y}{\sin \phi_{mn}} \quad (18.17)$$

To avoid grating lobes, the spacing between the elements must be less than λ ($d_y < \lambda$ and $d_x < \lambda$). In order a true grating lobe to occur, both equations (18.16) and (18.17) must have a real solution (θ_{mn}, ϕ_{mn}) .

3-D pattern of a 5-element square planar uniform array without grating lobes ($d = \lambda/4$, $\beta_x = \beta_y = 0$):

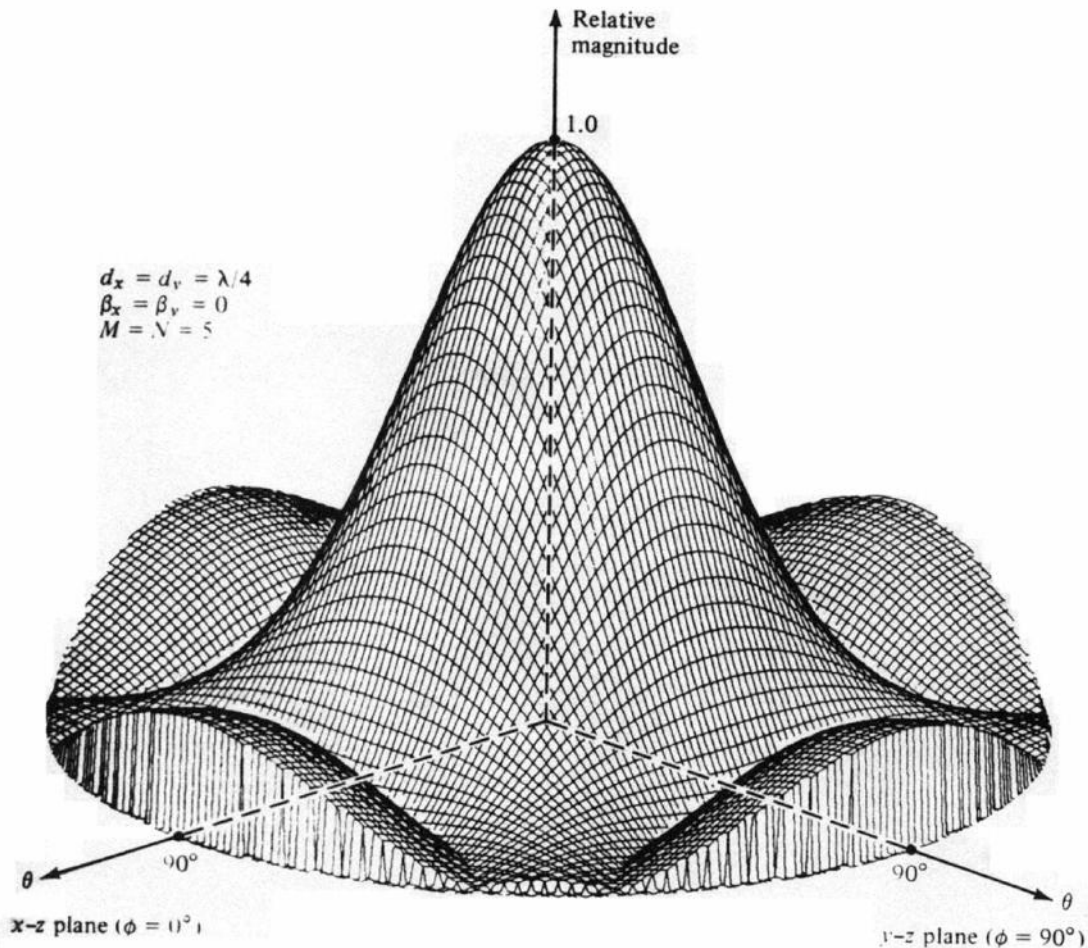


Fig. 6.24, pp.313 Balanis

3-D pattern of a 5-element square planar uniform array without grating lobes ($d = \lambda/2$, $\beta_x = \beta_y = 0$):

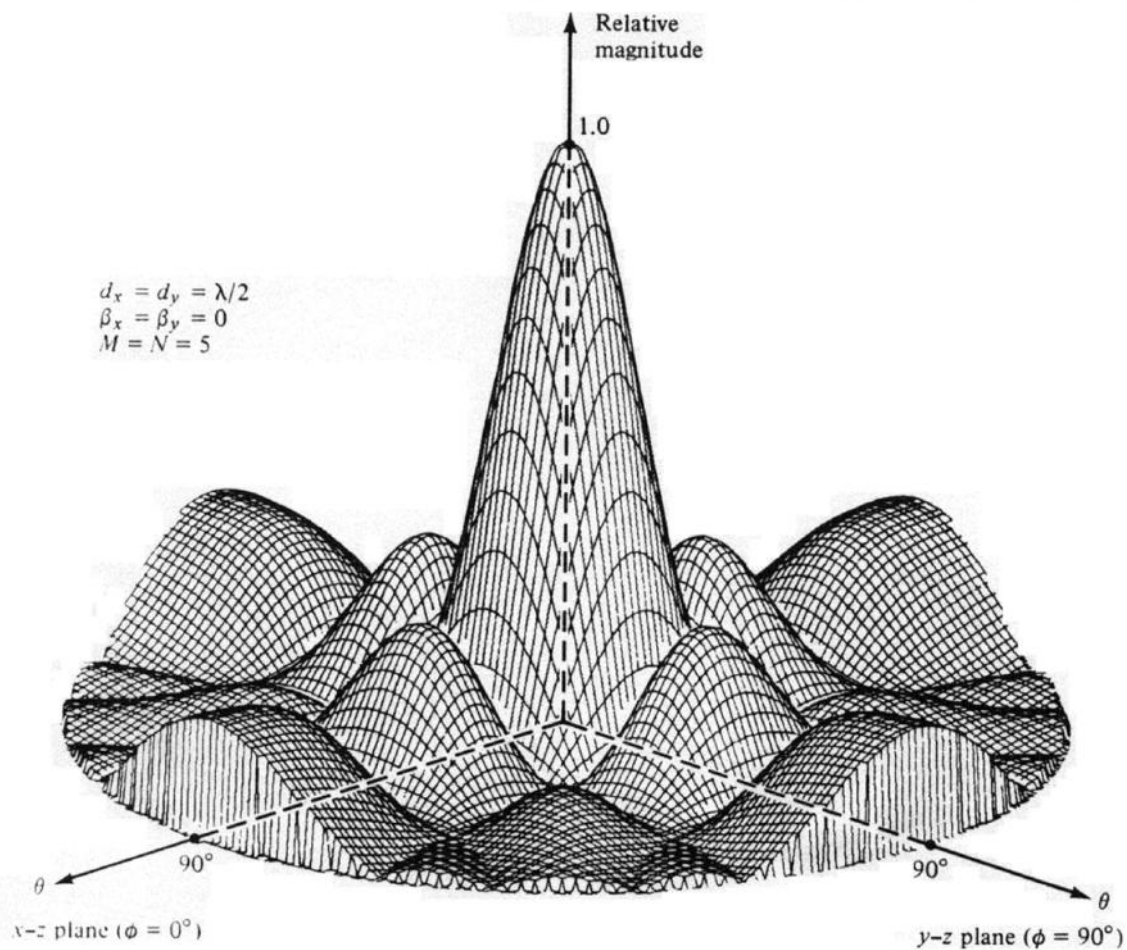
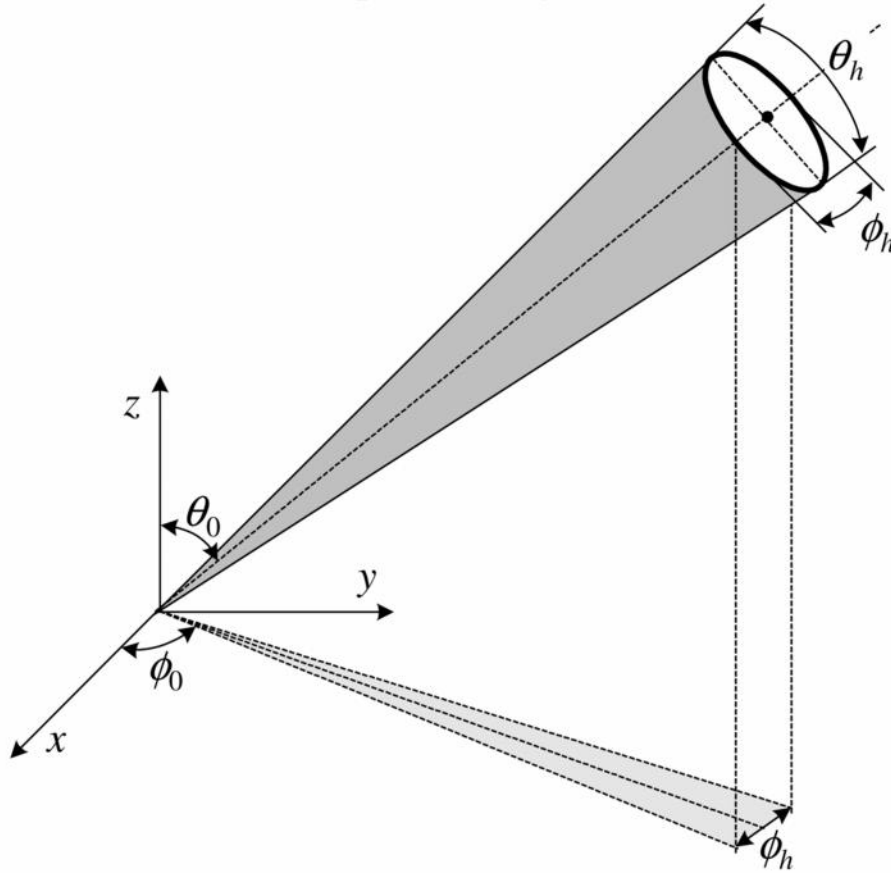


Fig. 6.25, pp.314, Balanis

Notice the considerable decrease in the beamwidth as the spacing is increased from $\lambda/4$ to $\lambda/2$.

1.2 The beamwidth of a planar array



A simple procedure, proposed by R.S. Elliot¹ will be outlined. It is based on the use of the beamwidths of the linear arrays building the planar array.

For a large array, whose maximum is near the broad side, the elevation plane HPBW is approximately:

$$\theta_h = \sqrt{\frac{1}{\cos^2 \theta_0 [\Delta \theta_x^{-2} \cos^2 \phi_0 + \Delta \theta_y^{-2} \sin^2 \phi_0]}} \quad (18.18)$$

¹ "Beamwidth and directivity of large scanning arrays", *The Microwave Journal*, Jan. 1964, pp.74-82

where: (θ_0, ϕ_0) specify the main-beam direction;

$\Delta\theta_x$ is the HPBW of a linear broadside array whose number of elements M and amplitude distribution is the same as that of the x -axis linear arrays building the planar array;

$\Delta\theta_y$ is the HPBW of a linear BSA whose number of elements N and amplitude distribution is the same as those of the y -axis linear arrays building the planar array.

The HPBW in the plane, which is orthogonal to the $\phi = \phi_0$ plane and contains the maximum, is:

$$\phi_h = \sqrt{\frac{1}{\Delta\theta_x^{-2} \sin^2 \phi_0 + \Delta\theta_y^{-2} \cos^2 \phi_0}} \quad (18.19)$$

For a square array ($M = N$) with amplitude distributions along the x and y axes of the same type, equations (18.18) and (18.19) reduce to:

$$\theta_h = \frac{\Delta\theta_x}{\cos \theta_0} = \frac{\Delta\theta_y}{\cos \theta_0} \quad (18.20)$$

$$\phi_h = \Delta\theta_x = \Delta\theta_y \quad (18.21)$$

From (18.20), it is obvious that the HPBW in the elevation plane very much depends on the elevation angle θ_0 of the main beam.

The HPBW in the azimuthal plane ϕ_h does not depend on the elevation angle θ_0 .

The beam solid angle of the planar array can be approximated by:

$$\Omega_A = \theta_h \phi_h \quad (18.22)$$

or

$$\Omega_A = \frac{\Delta\theta_x \Delta\theta_y}{\cos^2 \theta_0 \sqrt{\left[\sin^2 \phi_0 + \frac{\Delta\theta_y^2}{\Delta\theta_x^2} \cos^2 \phi_0 \right] \left[\sin^2 \phi_0 + \frac{\Delta\theta_x^2}{\Delta\theta_y^2} \cos^2 \phi_0 \right]}} \quad (18.23)$$

1.3 Directivity

The general expression for the calculation of the directivity of an array is:

$$D_0 = 4\pi \frac{|AF(\theta_0, \phi_0)|^2}{\int_0^{2\pi} \int_0^\pi |AF(\theta, \phi)|^2 \sin \theta d\theta d\phi} \quad (18.24)$$

For large planar arrays, which are nearly broadside, (18.24) reduces to:

$$D_0 = \pi D_x D_y \cos \theta_0 \quad (18.25)$$

where D_x is the directivity of the respective linear BSA, x -axis;
 D_y is the directivity of the respective linear BSA, y -axis.

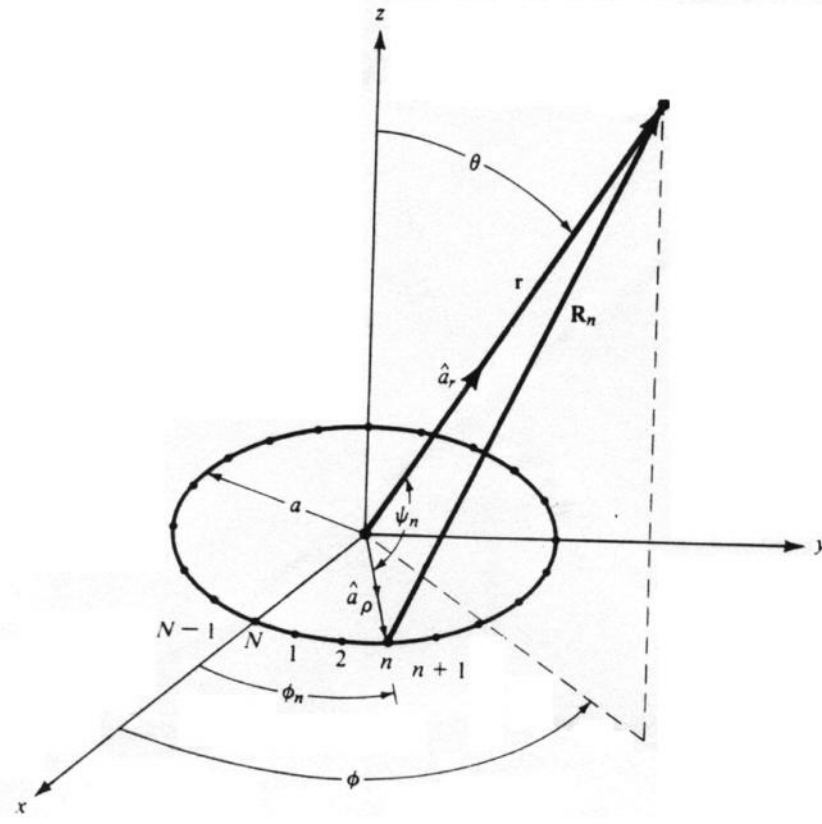
One can also use the array solid beam angle Ω_A in (18.23) to calculate the approximate directivity of a nearly broadside planar array:

$$D_0 \simeq \frac{\pi^2}{\Omega_{A[Sr]}} \simeq \frac{32400}{\Omega_{A[\text{deg}^2]}} \quad (18.26)$$

Remember:

- 1) The main beam direction is controlled through the phase shifts, β_x and β_y .
- 2) The beamwidth and side-lobe levels are controlled through the amplitude distribution.

2. Circular array



2.1 Array factor

The normalized field can be written as:

$$E(r, \theta, \phi) = \sum_{n=1}^N a_n \frac{e^{-jkR_n}}{R_n} \quad (18.27)$$

where:

$$R_n = \sqrt{r^2 + a^2 - 2ar \cos \psi_n} \quad (18.28)$$

For $r \gg a$, (18.28) reduces to:

$$R_n \simeq r - a \cos \psi_n \simeq r - a (\hat{a}_{\rho_n} \cdot \hat{r}) \quad (18.29)$$

In rectangular coordinate system:

$$\begin{cases} \hat{a}_{\rho_n} = \hat{x} \cos \phi_n + \hat{y} \sin \phi_n \\ \hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta \end{cases}$$

Therefore:

$$R_n = r - a \sin \theta (\cos \phi_n \cos \phi + \sin \phi_n \sin \phi) \quad (18.30)$$

Finally, R_n is approximated in the phase terms as:

$$R_n = r - a \sin \theta \cos(\phi - \phi_n) \quad (18.31)$$

For the amplitude term, the approximation

$$\frac{1}{R_n} \simeq \frac{1}{r}, \text{ all } n \quad (18.32)$$

is made.

Assuming the approximations (18.31) and (18.32) are valid, the far-zone array field is reduced to:

$$E(r, \theta, \phi) = \frac{e^{-jkr}}{r} \sum_{n=1}^N a_n e^{jka \sin \theta \cos(\phi - \phi_n)} \quad (18.33)$$

where: a_n is the excitation coefficient (amplitude and phase);

$$\phi_n = \frac{2\pi}{N} n \quad \text{is the angular position of the } n\text{-th element.}$$

In general, the excitation coefficient can be represented as:

$$a = I_n e^{j\alpha_n}, \quad (18.34)$$

where I_n is the amplitude term, and α_n is the phase of the excitation of the n -th element relative to a chosen array element of zero phase.

$$\Rightarrow E(r, \theta, \phi) = \frac{e^{-jkr}}{r} \sum_{n=1}^N I_n e^{j[ka \sin \theta \cos(\phi - \phi_n) + \alpha_n]} \quad (18.35)$$

The AF is obtained as:

$$AF(\theta, \phi) = \sum_{n=1}^N I_n e^{j[ka \sin \theta \cos(\phi - \phi_n) + \alpha_n]} \quad (18.36)$$

Expression (18.36) represents the AF of a circular array of N equispaced elements. The maximum of the AF occurs when all the phase terms in (18.36) equal unity, or:

$$ka \sin \theta \cos(\phi - \phi_n) + \alpha_n = 2m\pi, \quad m = 0, \pm 1, \pm 2, \text{ all } n \quad (18.37)$$

The principal maximum ($m = 0$) is defined by the direction (θ_0, ϕ_0) , for which:

$$\alpha_n = -ka \sin \theta_0 \cos(\phi_0 - \phi_n), \quad n = 1, 2, \dots, N \quad (18.38)$$

If a circular array is required to have maximum radiation in the direction (θ_0, ϕ_0) , then the phases of its excitations will have to fulfil (18.38). The AF of such an array is:

$$AF(\theta, \phi) = \sum_{n=1}^N I_n e^{jka[\sin \theta \cos(\phi - \phi_n) - \sin \theta_0 \cos(\phi_0 - \phi_n)]} \quad (18.39)$$

$$AF(\theta, \phi) = \sum_{n=1}^N I_n e^{jka(\cos \psi_n - \cos \psi_{0n})} \quad (18.40)$$

Here:

$\psi_n = \cos^{-1}[\sin \theta \cos(\phi - \phi_n)]$ is the angle between \hat{r} and \hat{a}_{ρ_n} ;
 $\psi_{0n} = \cos^{-1}[\sin \theta_0 \cos(\phi_0 - \phi_n)]$ is the angle between \hat{a}_{ρ_n} and \hat{r}_{\max}
 pointing in the direction of maximum radiation.

As the radius of the array a becomes very large as compared to λ , the directivity of the uniform circular array ($I_n = I_0$, all n) approaches the value of N .

Uniform circular array 3-D pattern ($N=10$, $ka = \frac{2\pi}{\lambda}a = 10$)

